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Magnetoresistance of potassium films in the Sondheimer geometry

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Abstract. We report a study of transverse magnetoresistance of thin, rolled potassium films. The measurements in the Sondheimer geometry, when the magnetic field is along the outwards normal to the plane of the film, yield no oscillatory pattern in the transverse magnetoresistance as a function of increasing field. For the high-field regime, with $\omega\tau =$ 10–30, there is a non-saturating linear increase in the magnetoresistance. The Kohler slope of this increase is dependent on the ratio of the film thickness to the electronic mean free path and is up to 20 times the appropriate slope for the bulk metal. From a comparison of these results with the data available for the MacDonald geometry (with the field lying in the film plane), we deduce that the origin of the magnetoresistance increase for the Sondheimer geometry is related to size effects. We discuss several possible explanations for the absence of Sondheimer oscillations, such as a large Hall field, open orbits due to the preferred orientation of the Q-domains associated with charge-density waves and inherent bulk magnetoresistance. None of them seems to provide a sufficient argument for the absence of the oscillatory pattern in the field dependence of the transverse magnetoresistance in the potassium thin films under study.

1. Introduction

The influence of size effects upon the electrical resistivity of thin, metallic films has been widely studied both theoretically and experimentally. Outstanding reviews of this field are presented by Brandli and Olsen (1969) and Tellier and Tosser (1982). It was found originally (Sondheimer 1950, MacDonald and Sarginson 1950) that the application of magnetic fields to samples with a characteristic dimension similar to the electronic mean free path (MFP) leads to new phenomena that are now frequently named magnetomorphic effects. Two field orientations in which to study the transverse magnetoresistance of thin films, named after their inventors, are the MacDonald and Sondheimer geometries. In each case, the field is perpendicular to the current density $\boldsymbol{J}$. For the MacDonald geometry it also lies in the plane of the film and in the Sondheimer geometry the field is oriented along the outwards normal to the plane. The mutual orientation of the magnetic field $\boldsymbol{B}$, the outwards normal to the plane of the film and the direction of the current density $\boldsymbol{J}$ is specified in figures 1(a) and 1(b) for the MacDonald and Sondheimer geometries, respectively. In what follows, we mark the Sondheimer and MacDonald geometry by the angle $\theta = [\boldsymbol{B} \wedge \boldsymbol{n}]$, with $\theta = 0°$ and $\theta = 90°$ in the former and the latter case, respectively. The predictions of the magnetoresistance behaviour in the MacDonald geometry were checked in early work with sodium by MacDonald and

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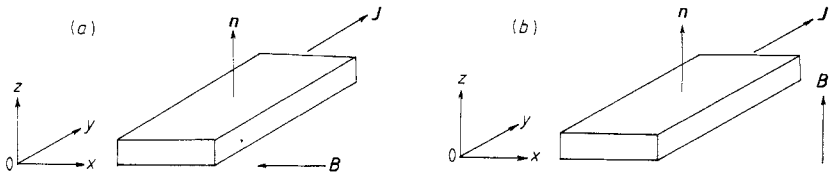

Figure 1. The mutual orientation of the magnetic field $\boldsymbol{B}$, the current density $\boldsymbol{J}$ and the outwards normal to the surface of the film sample $\boldsymbol{n}$ in the (*a*) MacDonald and (*b*) Sondheimer geometries.

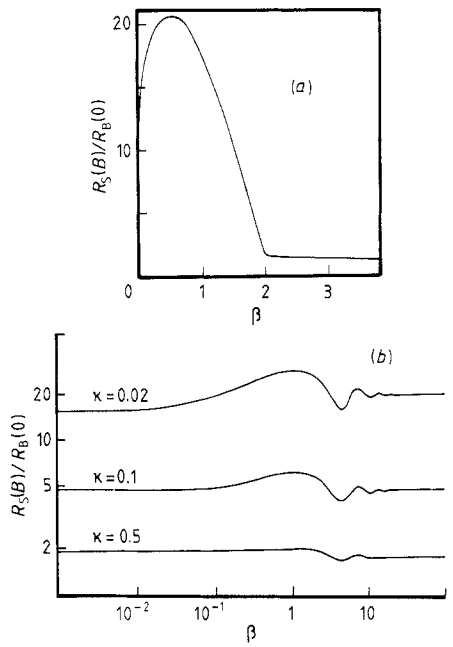

Figure 2. The schematically drawn qualitative behaviour of the ratio $R_S(B)/R_B(0)$ as a function of a variable β in the (*a*) MacDonald (see Ditlefsen and Lothe, figure 3) and (*b*) Sondheimer (see figure 2 in his paper) geometries ($p = 0$). Here $R_S(B)$ is the field-dependent resistivity of the sample, influenced by the presence of size effects, and $R_B(0)$ is the zero-field bulk resistivity of the sample material, both measured at $T = 4.2$ K. The variable $\beta = d/r$, i.e. (film thickness)/(maximum electronic orbit radius) is proportional to the field strength B.

Sarginson (1950), Chambers (1950) and White and Woods (1956). The oscillatory effects of the magnetomorphic origin predicted by Sondheimer for thin metallic films have been reported for sodium wires by Babiskin and Siebenmann (1957). We, however, found no reports of the size effects in thin potassium films with a thickness of tens of micrometres, which at 4.2 K is of the order of the MFP in pure bulk potassium.

The physical origin of the characteristic behaviour of magnetoresistance in the two geometries is quite different.

(i) *MacDonald geometry*; $\theta = 90°$. As shown schematically in figure 2(*a*), the ratio of the size-effect-dependent magnetoresistance to its free-electron bulk value at zero field shows an initial increase for low values of the variable $\beta = d/r = deB/mvc$, since some electrons suffer an initial decrease in MFP (MacDonald and Sarginson 1950; Azbel' 1963). Here d is the film thickness, r is the cyclotron radius and v is the electron velocity. There is a maximum at a field corresponding to $\beta = 0.55$ (Ditlefsen and Lothe 1966). In larger magnetic fields, the resistivity decreases with increasing field beyond the maximum because the effective electron free path of the spiral motion is increased by reducing the

collisions with the surface. For $\beta > 2.0$, the film thickness is greater than the diameter of the cyclotron orbit and the resistivity is expected from free-electron theory to be independent of magnetic field and to show saturation of size effects. This saturation might be affected slightly by the inherent bulk magnetoresistance which increases linearly in high fields (MacDonald and Sarginson 1950, Taub *et al* 1971).

(ii) *Sondheimer geometry*; $\theta = 0°$. The predicted (Sondheimer 1950) behaviour of the magnetoresistance in this geometry is shown schematically in figure 2(*b*). With the magnetic field pointing along the z axis (see figure 1(*b*)), the mean electronic velocity in this direction (Harrison 1960) is:

$$v_z = (-\hbar/2\pi m_c)(\partial A/\partial k_z)_{E=\text{constant}} \tag{1}$$

where A is the area of the orbit and m_c is the cyclotron mass.

To a first approximation under the condition $d \ll$ MFP, the time for the electrons to travel the film thickness d is $t_d = d/v_z$. Comparing this with the period T of the circular motion, one finds that an electron will complete $n = t_d/T$ orbits in crossing the film. Then, providing that the k-space orbits are closed, its net displacement in the film plane is zero. In this case, the net energy gain of the electron from the driving electric field is also zero. However, when $t_d/T = n + \frac{1}{2}$, the energy gain ΔE will be relatively large. Thus the ΔE is expected to be an oscillating function of magnetic field $\boldsymbol{B}$ with the period

$$|\Delta B| = (\hbar/ed)(\partial A/\partial k_z)_{E=\text{constant}}. \tag{2}$$

The periodic variation in ΔE provides a periodic change in the conductivity. The observed oscillations will come from the regions on the Fermi surface where the period is extremal or, as in the case of a spherical Fermi surface, where the plane k_z = constant just touches the Fermi surface and the orbit diameter shrinks to zero (Gurevich 1958). More generally, the observable periodic behaviour in the magnetoresistance will arise from the elliptic limiting points at which the plane k_z = constant is just tangent to the Fermi surface. Thus, the Gaussian curvature of the Fermi surface at the elliptic limiting points can be derived from the period of the Sondheimer oscillations (Gurevich 1958, Munarin and Marcus 1966). For the case of a cylindrical Fermi surface with $\boldsymbol{B}$ parallel to the cylinder axis, one will not have such a limiting point. Gurevich showed that the absence of an oscillatory behaviour provides information about the direction of open orbits if they are in a metal.

2. Experimental results

We used freshly cut potassium metal for the preparation of all the samples, excluding sample 1. The experiments were carried out at 4.2 K. The resistivity data were taken by the four-probe method with a computer control. The magnetic field was provided by an electromagnet with a maximum field of 1.8 T. More details of the sample preparation and the measuring procedure are given elsewhere (Gridin *et al* 1988). The various sample parameters are found in table 1. From the extrapolation to the bulk metal of the observed residual resistance ratios (RRR_S) *versus* the reciprocal sample thickness, one finds the RRR_B of the potassium metal and the MFP for each sample; the subscripts S and B stand for the size effect and bulk samples, respectively. We found $\text{RRR}_B = 2800$ and the corresponding MFP of samples 1, 3 and 5 was 81 μm, whereas those of samples 2 and 4 were 33 and 55 μm, respectively.

Table 1. Various parameters of the film samples studied in this work. Here κ is a film thickness MFP ratio; RRR_S is a residual resistance ratio for film samples; and $\omega\tau$ is evaluated from equation (4) at $B = 1.8$ T.

Sample No	R_S(295 K) (mΩ)	RRR_S	d (μm)	κ	$\omega\tau$
1	31.7	537	21.0	0.259	29.1
2	347.0	96	1.9	0.034	32.6
3	32.0	601	20.9	0.258	32.7
4	363.7	150	1.8	0.055	32.1
5	6.2	1562	108.0	1.333	30.5

The magnetic field orientation with respect to the outwards normal to the film surface was found from the angular dependence of the Hall effect. First, a rough estimate (to within 2–3°) was made of the angle θ(max) at which the maximum Hall voltage was obtained. Then, a 360° full angular dependence of the voltage was plotted versus cos θ, with the angle zero point at θ(max). The angular dependence of the Hall voltage for sample 3 is presented in figure 3. The point where the two straight lines cross in this figure provides the best orientation for the study of the MacDonald geometry ($\theta = 90°$). Rotating the field through 90° from this angle allows us to obtain the proper Sondheimer geometry, $\theta = 0°$. The results for the transverse magnetoresistance of sample 4

$$\Delta R_S(B)/R_S(0) = (R_S(B) - R_S(0))/R_S(0) \tag{3}$$

are plotted for both geometries in figure 4 as a function of $\omega\tau$, which is related to the field B (in T) by (Gridin *et al* 1988)

$$\omega\tau = 6.19 \times 10^{-3}\ RRR_S(1 + 1/\kappa)B \tag{4}$$

where $R_S(B)$ and $R_S(0)$ are the resistance measured with and without the field, respectively; κ is the film thickness/MFP ratio; the subscript S indicates the size effect case and RRR_S is the residual resistance ratio of our samples, defined by

$$RRR_S = R_S(295\ \text{K})/R_S\ (4.2\ \text{K}). \tag{5}$$

The R_S(295 K) and R_S(4.2 K) are the room temperature and the 4.2 K resistances of our samples, respectively (see table 1 for κ, RRR_S, R_S(295 K) and the range of $\omega\tau$ covered for each sample in this work).

Now we stress that reproducibility of results was achieved with the use of unoiled potassium metal for the sample preparation. All our samples, except 1, were rolled from freshly cut, unoiled potassium metal. Although the oil was thoroughly dried from sample 1, the small residual amount of oil on the film surface drastically affected the reproducibility of the results. This point is illustrated in figure 5, where we compare the $\theta = 90°$ data taken for samples 1 and 3 (figure 5(a)) and 2 and 4 (figure 5(b)), with each pair having similar κ-values.

The main feature of the field-dependent transverse magnetoresistance in the Sondheimer geometry was a linear increase for $\omega\tau > 10$. The Kohler slope of this linear dependence was found to be an increasing function of increasing κ. Small deviations from linearity were observed at $\omega\tau < 10$.

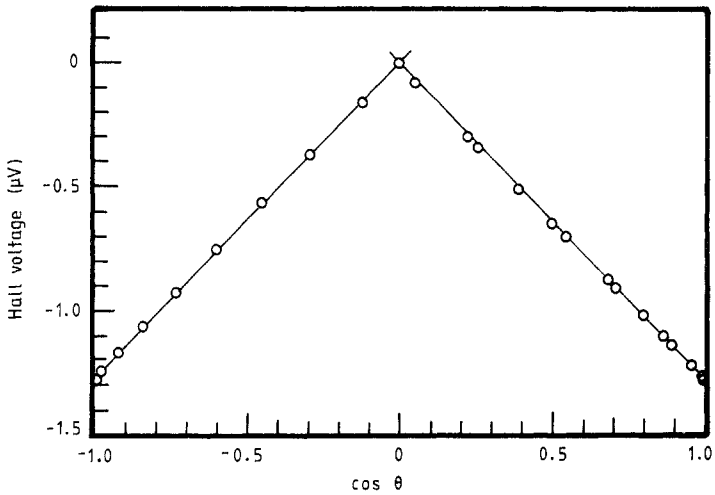

Figure 3. The angular dependence of the Hall voltage of sample 3. The best field orientation for the MacDonald geometry corresponds to the angle $\theta = \boldsymbol{B}\hat{}\boldsymbol{n}$, which is determined by the point where the two straight lines cross.

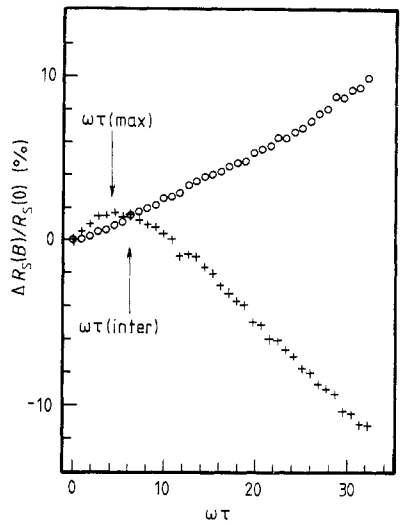

Figure 4. The transverse magnetoresistance of sample 4 in the MacDonald (+) and Sondheimer (○) geometries as a function of $\omega\tau$. See equations (7) and (8) for the κ-dependence of the position of the maximum, $\omega\tau(\max)$, in the magnetoresistance studied in the MacDonald geometry and the κ-dependence of the intersection point, $\omega\tau(\text{inter})$.

According to theoretical predictions for this case (Sondheimer 1950), there should be a slight initial increase in magnetoresistance, followed by a first maximum at $\beta = d/r = 1$ (or $\omega\tau = \beta/\kappa = 1/\kappa$). Then a quickly decaying oscillatory pattern should appear with the period of $\Delta\beta = 6$. In terms of $\omega\tau$, we expect to see the first maximum at $\omega\tau = 29.4, 18.2, 3.9$ and 0.8 for samples 2, 4, 3 and 5, respectively. In figure 6, we plot the field dependence of the transverse magnetoresistance of samples 3 and 5 (for sample 4, see figure 4; sample 2 gave essentially the same behaviour as that of sample 4). One sees

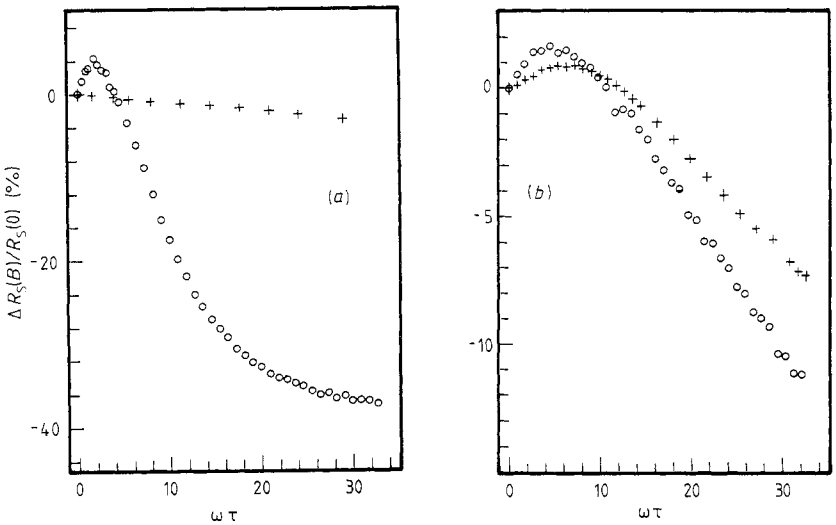

Figure 5. The transverse magnetoresistance in the MacDonald geometry of: (*a*) samples 1 (+) and 3 (○) and (*b*) 2 (+) and 4 (○). The samples in each pair have similar κ-values. Sample 1, however, which was prepared from oiled potassium does not show the characteristic feature of field-dependent resistivity in this geometry.

that there is no evidence of the first maximum and of the multiple maxima. The evidence of size-related effects in this geometry was the saturation-like κ-dependence of the Kohler slope, S, which is shown in figure 7. The values of the positive Kohler slopes found at $\theta = 0°$ are given in table 2. We note that, for instance, the slope of the thickest sample, sample 5, is more than 20 times larger than the reported value of 0.004 for bulk potassium wires ($\simeq$2 mm) from metal with the same purity with $RRR_B = 2500$–3000 (Taub *et al* 1971).

In order to show that the observations in the Sondheimer geometry have a size-effect-related origin, we make use of some of the data collected for the MacDonald geometry (Gridin *et al* 1988).

The positive Kohler slopes in the Sondheimer geometry are closely related to the negative slopes observed for the MacDonald field orientation. As is seen in figures 4 and 5, there is a field region immediately after the initial maximum where the magnetoresistance shows a linear decrease, with quite a large negative Kohler slope. We found, similarly to in the case of the $\theta = 0°$ geometry, that the magnitude of this slope is dependent on κ. This dependence is presented in figure 7 and the absolute values are given in table 2. We found a close correspondence between the magnitude of the positive and negative slopes at $\theta = 0°$ and $\theta = 90°$, respectively. One sees in figure 7 that for $\kappa < 0.26$ the functional dependence on κ of the Kohler slopes observed in the $\theta = 90°$ geometry is the square root of that for the $\theta = 0°$ one, with

$$\left.\begin{aligned} S(\kappa;\, \theta = 90°) &\sim \kappa^1 \\ S(\kappa;\, \theta = 0°) &\sim \kappa^2 \end{aligned}\right\} \qquad \kappa < 0.26 \tag{6}$$

where we use an explicit labelling for the dependences on κ and angle of the Kohler slopes, $S(\kappa;\, \theta)$. The deviation from the behaviour when $\kappa > 0.26$ can be understood qualitatively. In the proximity of bulk magnetoresistance effects the Kohler slope is as

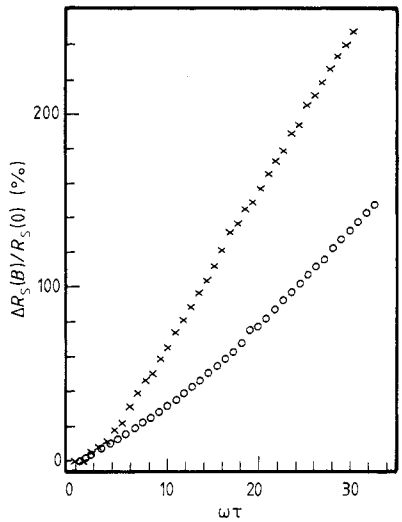

Figure 6. The transverse magnetoresistances of sample 3 (○) and sample 5 (+) measured in the Sondheimer geometry.

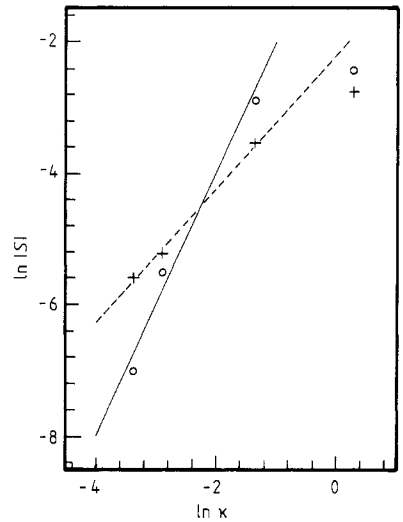

Figure 7. The κ-dependence of the Kohler slopes, S, in the MacDonald (+) and Sondheimer (○) geometry plotted on natural logarithmic scales to illustrate the linear and quadratic power laws for the κ-dependences of Kohler slopes measured in the MacDonald and Sondheimer geometries, respectively, for $\kappa < 0.26$; see equation (6) in the text.

low as $S(\infty) = 0.004$ (Taub *et al* 1971) for all field directions in polycrystalline samples.

According to the typical behaviour of the transverse magnetoresistance, as shown in figures 1(*a*) and 1(*b*), there should exist an intersection point when the field dependences of the magnetoresistances measured in the Sondheimer and MacDonald geometries are brought together in the same plot. Our findings support this expectation for samples 2, 4, 3 and 5. We show this intersection point in figure 4 for sample 4. In fact, we found that the $\omega\tau$ position of this point is inversely proportional to κ. In figure 8 this observation is shown by a broken line, representing the empirical relation of the $\omega\tau$(inter) versus $1/\kappa$ via

$$\omega\tau(\text{inter}) = 0.32/\kappa. \tag{7}$$

In figure 8 we also show the theoretical prediction (Ditlefsen and Lothe 1966) for the

Table 2. Absolute value of the Kohler slopes, $S = S(\kappa)$, as measured for the Sondheimer ($\theta = 0°$) and MacDonald ($\theta = 90°$) geometries.

Sample No	Positive Kohler slope at $\theta = 0°$	Negative Kohler slope at $\theta = 90°$
2	1×10^{-3}	4×10^{-3}
3	56×10^{-3}	29×10^{-3}
4	4×10^{-3}	6×10^{-3}
5	89×10^{-3}	64×10^{-3}

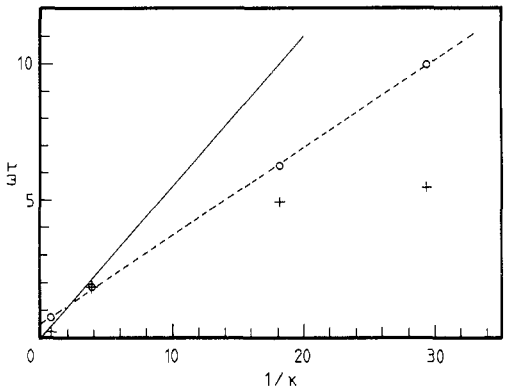

Figure 8. The values of $\omega\tau$(max) (○) and $\omega\tau$(inter) (+) versus $1/\kappa$. See figure 4 for the definitions of $\omega\tau$(max) and $\omega\tau$(inter). The full line is the theoretical prediction for the κ-dependence of $\omega\tau$(max) (see equation (8)); the broken line is the empirical fit, given by equation (7), to the $\omega\tau$(inter) data.

position of the initial maximum

$$\omega\tau(\text{max}) = 0.55/\kappa \tag{8}$$

as a full line together with the experimental values of $\omega\tau$(max) defined in figure 4. One sees that, apart from the prefactors in equations (7) and (8), the $1/\kappa$ dependence fits the observation for the $\omega\tau$ of the intersection point better than that for the position of the maximum. One sees that for the thicker samples 3 and 5 the $\omega\tau$(inter) essentially coincides with $\omega\tau$(max), but as κ decreases the intersection occurs at higher $\omega\tau$ than that for which the magnetoresistance reaches its maximum.

3. Discussion

According to Sondheimer's prediction, the potassium samples 4, 3 and 5 are capable of giving, respectively, at least one, two and seven oscillations in the transverse magnetoresistance within the range of $\omega\tau$ covered. It should be noted, however, that for an uncompensated metal such as potassium, the amplitude of the oscillations decays like B^{-4}, according to a theoretical study of this case (Gurevich 1958). Thus one does not expect to observe a multiple oscillatory pattern. In our study, we observed no evidence of even one maximum for this geometry. This is similar to the observation of MacDonald, who found a steady increase in magnetoresistance as a function of increasing field strength for sodium and rubidium (MacDonald 1957). Since all our measurements were carried out with the reversal of current and magnetic field direction, the data reported here do not include thermal and Hall voltage components. The difference between the potential drops at opposite fields, before the summing (a standard procedure to eliminate the Hall component), was as low as 0.1%, 1.5% and 10% at $B = 1.8$ T for samples 2 (4), 3 and 5, respectively. In view of the observed degree of resistance increase, as shown in figures 4 and 6, this is a quite negligible effect even if one has not carried out the $(+B)$ and $(-B)$ measurements. We thus conclude that the sample shape argument (Cotti

1961), as an explanation of the absence of the Sondheimer oscillations, does not hold in our study.

Another possible explanation of the absence of an oscillatory pattern arises from the case considered by Overhauser (1971). According to his theoretical predictions the charge-density wave Q-domains have their preferred orientation along the [110] direction. It was found (Monin and Boutry 1974) that potassium films grown on smooth amorphous substrates have the [110] axis perpendicular to the surface. For the purposes of discussion, we assume that this might hold for the rolled films as well and provide open orbits in this direction. Then, there should not be (Gurevich 1958) an oscillatory behaviour when the field is parallel to the film's normal, i.e. in the Sondheimer geometry.

Our argument against this scenario is twofold. First, the cooling–warming cycling should cause a reorientation of Q-domains. This is frequently used by Overhauser (1978) for an explanation of the considerable lack of reproducibility in experimental studies with potassium. We found that the results were stable against thermal cycling within 1% provided that freshly cut, unoiled metal was used for the sample preparation. Secondly, the alignment of the $\boldsymbol{Q}$ vector with the film's normal should produce a non-linearity in the angular dependence of the Hall coefficient (Huberman 1987), when the latter is plotted as a function of $\cos\theta$. As shown in figure 3, this was not observed in our study. We note that for the absence of the observable oscillatory pattern it is sufficient that the z component of the velocity of conducting electrons, v_z, is very small, i.e. $v_z \ll v_F$. This condition (less restrictive than for the existence of open orbits) yields a very small period of oscillations in terms of ΔB (see equation (2)) which together with their rapidly decaying amplitude is just enough to prevent their experimental observation.

Let us consider the possibility that an inherent bulk magnetoresistance of potassium is responsible for the absence of the weak (decaying like B^{-4}) oscillatory pattern. According to the theory developed for vanishing bulk magnetoresistance (Sondheimer 1950) in a 'free-electron' metal like potassium, the amplitude of the first oscillation can be as large as 20% of the zero-field resistivity when $\kappa = 0.5$ and it increases with decreasing κ. Thus, for instance, for sample 3 with $\kappa = 0.26$ we should observe at least the first maximum in $\Delta R_S(B)/R_S(0)$.

With the mean accuracy of our data up to 0.25% and bulk values of Kohler slopes available from the study of Taub *et al* (1971), $S = 0.004$, we therefore rule out the masking of an oscillatory pattern by the bulk magnetoresistance. On the other hand, it appears that the positive Kohler slopes in the $\theta = 0°$ geometry are closely related to the negative ones observed at $\theta = 90°$ (see figure 7 and the discussion above). In order to establish the first maximum, one needs to cover the $\omega\tau$ region of $\Delta\omega\tau = (3\text{–}6)/\kappa$ (or $\Delta\beta = 3\text{–}6$; see paragraph I, case $\theta = 0°$, above).

For sample 3, for instance, with a Kohler slope $S = 0.056$ we get a linear increase of resistance for this $\Delta\omega\tau$ region that is as large as 70–130%. This increase is enough to prevent the clear observation of the oscillatory pattern. We, therefore, come to the conclusion that the size-dependent enhancement in the Kohler slopes serves by itself as the cause of the absence in the observable oscillations in magnetoresistance at $\theta = 0°$, i.e. for the Sondheimer geometry.

In other words, we found that the size-dependent enhancement of the Kohler slope at high fields is a primary effect of the confined electronic motion in samples with $d <$ MFP, whereas the oscillatory pattern is probably only the secondary one, which can be masked by the large linear increase in the magnetoresistance at $\theta = 0°$.

Let us discuss now the κ-dependent increase in the magnetoresistance for the Sondheimer geometry. It was shown by Sondheimer that thin films of metals, which do

not exhibit bulk magnetoresistance, will show size-effect-dependent magnetoresistance because of the constraint on the motion of carriers due to the reduced thickness. Sondheimer has found that in the limit of very small magnetic fields and small κ-values the magnetoresistance $\Delta R(B)/R(0)$ goes to zero. Jain and Verma (1973) have extended this analysis to the case of small fields and arbitrary κ-values. They found (Jain and Verma (1973), equation (20)) that

$$\Delta R(B)/R(0) \sim B^2 Q(\kappa). \tag{9}$$

The κ-dependence of the magnetoresistance is described by the function $Q(\kappa)$, which increases with κ varying from 0.01 to 1 and after reaching a maximum at $\kappa = 1$ (assuming diffusive scattering, $p = 0$) decreases to zero, for $\kappa \to \infty$. We found that the deviation from a linear increase in magnetoresistance occurs for $\omega\tau < 10$ and does not agree with the quadratic field dependence, given by equation (9), for $\omega\tau > 10$. We now check the theoretical prediction for the κ-dependence of the $\Delta R(B)/R(0)$, which is implicitly given by $Q(\kappa)$ in equation (9). Using the observed linearity in $\Delta R_S(B)/R_S(0)$ for $\omega\tau > 10$, we write

$$\Delta R_S(B)/R_S(0) \sim \omega\tau \qquad S(\kappa),\ \omega\tau > 10 \tag{10}$$

where $S(\kappa)$ is the κ-dependent (or sample-dependent) Kohler slope of the field dependence (through $\omega\tau$) of the transverse magnetoresistance in the Sondheimer geometry. Since the transition from a non-linear (low-$\omega\tau$) towards a linear (high-$\omega\tau$) field dependence in the magnetoresistance occurs in a smooth (see figures 4 and 6) manner, we can write

$$Q(\kappa) \sim S(\kappa). \tag{11}$$

Assuming a totally diffusive scattering, i.e. $p = 0$, the κ-dependence of $Q(\kappa)$ is given (Jain and Verma 1973) by

$$Q(\kappa) = \Phi(\kappa)\,(1 - \Phi(\kappa)/F(\kappa))$$

$$\Phi(\kappa) = 2F(\kappa) - 1 + R(\kappa)$$

$$F(\kappa) = 1 - \frac{3}{8\kappa} + \frac{3}{2}\kappa \int_{\kappa}^{\infty} \exp(-r) \left(\frac{1}{r}\right)^3 \left[1 - \left(\frac{\kappa}{r}\right)^2\right] \mathrm{d}r$$

$$R(\kappa) = \frac{3}{2}\kappa \int_{\kappa}^{\infty} \exp(-r) \left(\frac{1}{r}\right)^2 \left[1 - \left(\frac{\kappa}{r}\right)^2\right] \mathrm{d}r. \tag{12}$$

To return to the original forms for $F(\kappa)$ and $R(\kappa)$, which are given by Jain and Verma, one just substitutes for the variable r in equation (12) with $\kappa/\cos\theta$. (We note that there is an obvious misprint in equation (14) of the Jain and Verma paper: the right-hand side (RHS) should be 1 − RHS). In figure 9 we present, on natural logarithmic scales, the calculated $Q(\kappa)$ of equation (12) together with our $S(\kappa)$ data (see table 2 for the values of Kohler slopes for different samples in the Sondheimer geometry) and the available value of $S = 0.004$ for bulk (2 mm thick) potassium wire (this corresponds to $\kappa = 25$, assuming MFP = 80 μm), provided by Taub *et al* (1971). One sees that there is a good qualitative agreement with the theoretical predictions of Jain and Verma for the existence of a maximum at $\kappa = 1$ in the κ-dependence of the magnetoresistance in the Sondheimer geometry.

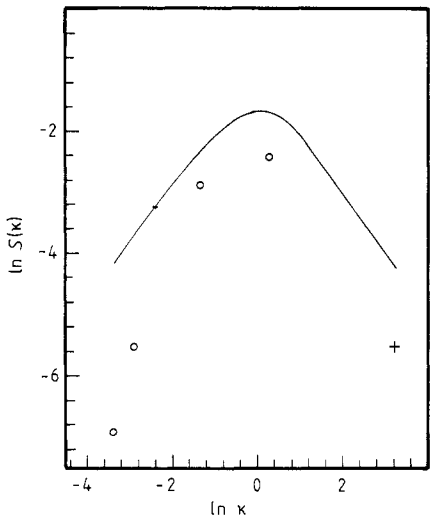

Figure 9. A plot of the κ-dependent Kohler slopes, $S(\kappa)$, in the Sondheimer geometry from (○) this work, and (+) that of Taub *et al* (1971). The full curve is the plot of $Q(\kappa)$, which is given by equation (12). We note a good qualitative agreement for the existence of a maximum in $Q(\kappa)$ at $\kappa = 1$ (Jain and Verma 1973).

4. Conclusions

We found that a high reproducibility of results can be achieved when thin potassium films are prepared from freshly cut, unoiled potassium metal. The measurements in the Sondheimer geometry gave no observable oscillatory pattern in the field dependence of the transverse magnetoresistance. On the other hand, for the high-field regime, with $\omega\tau = 10$–30, there is a steady non-saturating linear increase in the magnetoresistance. We found a good qualitative agreement between the κ-dependence of the Kohler slope of this increase and the available theoretical analysis of the magnetoresistance in the Sondheimer geometry. This size-dependent enhancement of the Kohler slopes at high fields is found to be sufficient to provide an effective 'masking' of the oscillatory part of the field-dependent magnetoresistance in the Sondheimer geometry. However, the absence of the oscillatory pattern can be explained alternatively by an unjustified assumption of a very small component of an electronic velocity in the direction of the normal to the film plane. The field-dependent reduction of the normal component of the electronic velocity is readily understood in the MacDonald geometry, since with the field parallel to the film surface, the probability that an electron strikes the external surfaces decreases with the increasing field strength.

In the Sondheimer geometry, however, this component is directed along the magnetic field. One therefore needs a field-independent mechanism for the reduction of the electronic velocity in the normal direction.

We conclude that in view of the present study and the more detailed analysis of the MacDonald geometry (Gridin *et al* 1988) for which we found only qualitative agreement with the existing models of size effects in a 'free-electron' metal, a revision of the theory is desirable. It should be noted that investigation of size effects in potassium is also of fundamental importance for the checking of the conceptual grounds underlying the more complicated modern theories of condensed matter.

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References

Azbel' M Ya 1963 *J. Exp. Theor. Phys.* **44** 1262 (Engl. Transl. 1963 *Sov. Phys.–JETP* **17** 851)
Babiskin J and Siebenmann P G 1957 *Phys. Rev.* **107** 1249
Brandli G and Olsen J L 1969 *Mater. Sci. Eng.* **4** 61
Chambers R G 1950 *Proc. R. Soc.* A **202** 378
Cotti P 1961 *Helv. Phys. Acta* **34** 777
Ditlefsen E and Lothe J 1966 *Phil. Mag.* **14** 759
Gridin V V, Datars W R and Ning Y B 1988 *Phys. Rev.* B at press
Gurevich V L 1958 *J. Exp. Theor. Phys.* **35** 668 (Engl. Transl. 1959 *Sov. Phys.–JETP* **35** 464)
Harrison W A 1960 *Phys. Rev.* **118** 1190
Huberman M L 1987 *Phys. Rev.* B **35** 8969
Jain G C and Verma B S 1973 *Thin Solid Films* **15** 115
MacDonald D K C 1957 *Phil. Mag.* **2** 97
MacDonald D K C and Sarginson K 1950 *Proc. R. Soc.* A **203** 223
Monin J and Boutry G A 1974 *Phys. Rev.* B **9** 1309
Munarin J A and Marcus J A 1966 *Bull. Am. Phys. Soc.* **11** 170
Overhauser A W 1971 *Phys. Rev.* B **3** 3173
—— 1978 *Adv. Phys.* **27** 343
Sondheimer E H 1950 *Phys. Rev.* **80** 401
Taub H, Schmidt R L, Maxfield B W and Bowers R 1971 *Phys. Rev.* B **4** 1134
Tellier C R and Tosser A J 1982 *Size Effects in Thin Films* (Amsterdam: Elsevier Scientific)
White G K and Woods S B 1956 *Phil. Mag.* **1** 846